Asymptotic Methods in Turbulent Combustion

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Introduction

THERE are a number of different regimes of turbulent combustion, dependent upon the intensity and scales of the turbulence, measured with respect to suitable combustion parameters derived from the chemical reaction rates. In one set of these regimes, combustion occurs in thin reaction sheets, transported and distorted by the turbulence. In these reaction sheet regimes, more than one characteristic length scale is involved in the turbulent combustion; there are short scales associated with the chemical processes and long scales associated with the turbulence. The disparity of scales causes asymptotic methods to be advantageous for studying turbulent combustion in reaction sheet regimes. A significant amount of progress has been made recently by use of asymptotic methods for describing these regimes in both premixed and nonpremixed turbulent combustion. By and large, the objectives have been not to calculate the turbulent reacting flows completely, but rather to relate the properties of interest in these flows to properties of nonreacting turbulent flows. It then becomes possible to use the existing methods of analysis of nonreacting flows to calculate the results of interest for turbulent combustion. The intent of the present paper is to review the recent advances achieved by use of the methods described above and to identify not only what is known but also areas of unknowns for future research. Other reviews covering material of this type have been published.[1-8]

The techniques employed in analyzing turbulent combustion differ for premixed and nonpremixed systems. This is especially true in various finer details of analyses of reaction sheet regimes. Therefore, it will be convenient here to treat turbulent premixed flames and turbulent diffusion flames separately. Presentations more unified in character may become appropriate in the future since there are a number of similarities, e.g., the reaction sheet aspect itself. However, it seems likely that certain essential differences will remain; burning velocities exist for premixed but not for nonpremixed combustion (at least not in the same sense).

Turbulent Premixed Flames

Combustion Regimes

Premixed laminar flames are characterized by a laminar burning velocity that can be expressed very roughly as $v_0 = \sqrt{D_L/\tau_c}$, where D_L is a laminar diffusivity and τ_c a representative chemical conversion time in the hot part of the laminar flame. A nondimensional measure of the intensity of turbulence for premixed turbulent combustion is the ratio of the root mean square velocity fluctuation to the laminar burning velocity, viz., $\sqrt{2q}/v_0$, where q is the turbulent kinetic energy in the unburned gas. A laminar flame length against which the turbulence scales can be measured is the flame thickness, $\delta = D_L/v_0$. Regimes of premixed turbulent combustion can be considered in a parameter plane whose coordinates are the nondimensional intensity and the ratio of a scale of turbulence to δ. Figure 1 illustrates this parameter plane, with the Kolmogorov scale ℓ_k selected for the turbulence length. A plot equivalent to Fig. 1, but with a different selection of coordinates, was given by Bray.[9]

For a rough classification of regimes in reacting gases, all molecular diffusivities may be set equal, so D_L is the same as the kinematic viscosity ν. It may then be demonstrated that the turbulence Reynolds number based on the integral scale ℓ is[4] $R_\ell \equiv \sqrt{2q}\ell/\nu = [(\sqrt{2q}/v_0)(\ell_k/\delta)]$, a relationship that enables lines of constant values of R_ℓ to be plotted as shown in Fig. 1. Turbulence Reynolds numbers $R_t \equiv \sqrt{2q}\ell_t/\nu$, based on the Taylor scale ℓ_t, are directly related to R_ℓ and are also indicated in Fig. 1, as is the ratio of the integral scale to the laminar flame thickness.

Ratios of flow or transport times to chemical times are Damköhler numbers that are of relevance in identifying regimes of reacting flows. One definition of a Damköhler number is the ratio of a large-eddy time, $\ell/\sqrt{2q}$, to τ_c; lines of constant values of this quantity are shown in Fig. 1. Another definition would be the ratio of a Kolmogorov eddy time to τ_c. In terms of the dissipation rate ϵ the characteristic time for a Kolmogorov eddy is $(\nu/\epsilon)^{1/2} = [D_L\ell/(2q)^{3/2}]^{1/2}$. The Damköhler number based on this time can be shown to be (ℓ_k/δ), whence vertical lines in Fig. 1 correspond to constant values of this Damköhler number.

For $R_\ell < 1$ and to the left of the vertical line labeled $\ell/\delta = 1$ in Fig. 1 (i.e., in the part of the diagram identified as "decay of turbulence") a representative decay time for turbulence is less than the transit time through a laminar flame. This regime is of little interest and will not be considered further here. Two important limiting regimes in Fig. 1 are those of distributed reactions and of reaction sheets. In the former the turbulent mixing is rapid compared with the chemistry, thereby causing the combustion to occur in a distributed zone, and in the latter the chemistry in hot regions is rapid compared with the turbulent processes, so that the combustion occurs in thin sheets wrinkled by the turbulence. It is in the latter regime that asymptotic methods have been useful.

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Presented as Paper 84-0475 at the AIAA 22nd Aerospace Sciences Meeting, Reno, NV, Jan. 9-12, 1984; received Dec. 3, 1984; revision submitted May 8, 1985. Most of this material is abstracted and condensed from the author's book, *Combustion Theory* (2nd ed.), Benjamin/Cummings, Menlo Park, CA, 1985, Chapter 10.

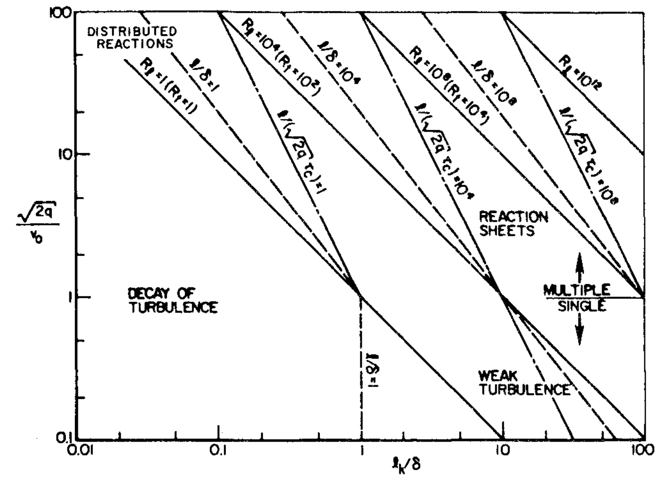

Fig. 1 Parameter plane of nondimensional intensity and nondimensional Kolmogorov scale of turbulence for premixed turbulent combustion, showing regimes of combustion and lines for constant values of turbulence Reynolds numbers, nondimensional integral scale, and a Damköhler number.

Although the two limiting regimes exist, the precise location of the boundary separating them is not well understood. Damköhler[10] first recognized the two regimes and suggested that $\ell/\delta = 1$ may be an appropriate boundary. If this is true, then the reaction sheet regime extends farther than it would if a criterion based on a large-eddy Damköhler number of unity were to apply. Kovasznay[11] offered the alternative boundary criterion that $v_0/\delta = \sqrt{2q}/\ell_t$, which is equivalent to $(\ell_k/\delta)^2 = 1$, thereby giving an even narrower reaction sheet regime (the right half of Fig. 1). Since δ/v_0 is a residence time in a laminar flame, Kovasznay's criterion can be interpreted as a requirement that the product of the residence time with the representative strain rate $(\sqrt{2q}/\ell_t)$ of the turbulent field be unity.[12] It seems clear that $\delta \ll \ell_k$ is at least a sufficient condition for occurrence of the reaction sheet regime and $\delta \gg \ell$ a sufficient criterion for the regime of distributed reaction. It is possible that in $\ell_k \ll \delta \ll \ell$ one or more intermediate regimes occur with distinguished characteristics of their own.

Within a limiting regime, dominant attributes of the turbulent combustion may vary with the parameters. In the reaction sheet regime, highly convoluted, folded, and multiply connected reaction sheets may occur at high turbulence intensities and weakly wrinkled, simply connected sheets in the portion of Fig. 1 marked "weak turbulence." Important ideas concerning flame structures and flame speeds in the subregime of multiple sheets[13,14] have found relatively little use for asymptotic methods. Therefore, the present discussion is focused mainly on the subregime of single sheets. A thorough review of this subregime is available.[8]

The boundary between the subregime of multiple and single sheets has been suggested[14] to be approximately $\sqrt{2q}/v_0 = 1$, as indicated in Fig. 1. However, the position of this boundary is uncertain. Unless ℓ_k/δ is sufficiently large, diffusive thermal effects on the laminar flame structure cause the boundary location to depend on this ratio and therefore to depart from a horizontal line in Fig. 1. Even at very large values of ℓ_k/δ, hydrodynamic instability of premixed laminar flames may cause the level of a horizontal boundary to vary with heat release. At large values of ℓ_k/δ, the laminar flame is of negligible thickness and therefore characterized solely by v_0, but at a fixed value of $\sqrt{2q}/v_0$ the variation of R_ℓ with ℓ_k/δ may cause the tendency toward multiply connected sheets to vary with this parameter; the value of $\sqrt{2q}/v_0$ at the boundary might be expected to decrease slightly with increasing ℓ_k/δ because of the increasing range of turbulence scales.

Turbulent Burning Velocity

In the reaction sheet regime, it is natural to relate the turbulent burning velocity v_T to the shape of the reaction sheet. This is simplest at values of ℓ_k/δ sufficiently large for the structure of the wrinkled laminar flame to be affected negligibly by turbulence, so that the constant laminar burning velocity v_0 applies to each point on the reaction sheet. As first observed by Damköhler,[10] since all of the fluid flowing through the turbulent combustion flame also passes through the wrinkled laminar flame, it is necessary, by consideration of the overall mass balance, that on the average $\rho_0 v_T a = \rho_0 v_0 a_f$, where a and a_f are the cross-sectional area of the turbulent flow and the average area of the wrinkled flame sheet, respectively, and ρ_0 is the density of the fresh mixture. Thus,

$$v_T = v_0 (a_f/a) \tag{1}$$

There is a general way to write the area ratio a_f/a of Eq. (1) in terms of a function $G(x,t)$, defined so that $G(x,t) = 0$ on the reaction sheet. Let $\boldsymbol{e}$ be a constant unit vector pointing in the direction of propagation of the turbulent flame. Then, at any given point on the reaction sheet, the ratio of an area element of the sheet to the projection of that area element on the direction of propagation is $|\nabla G| / |\boldsymbol{e} \cdot \nabla G|$. Therefore, with an overbar denoting a suitable ensemble average,

$$v_T = v_0 \left(\overline{|\nabla G| / |\boldsymbol{e} \cdot \nabla G|} \right)_{G=0} \tag{2}$$

If the reaction sheet nowhere becomes parallel to the vector $\boldsymbol{e}$, then its location everywhere may be described by putting $G(x,t) = x - F(y,z,t)$, where $x = (x,y,z)$ and x is a coordinate in the propagation direction $\boldsymbol{e}$ of the turbulent flame. With this selection, $|\boldsymbol{e} \cdot \nabla G| = 1$ and $|\nabla G| = \sqrt{1 + |\nabla F|^2}$, so that Eq. (2) reduces to

$$v_T = v_0 \overline{\sqrt{1 + |\nabla F|^2}} \tag{3}$$

in which only the transverse gradients of F are nonvanishing.

These results do not solve the problem of calculating v_T because it is necessary to relate the average area ratio to properties of the incoming turbulence. For turbulent flows that are statistically stationary and homogeneous in transverse directions (y and z), in a first approximation for small transverse gradients of F (necessitating large values of ℓ_k/δ), the relationship[15,16]

$$\partial F / \partial t = v' \tag{4}$$

applies, where v' is the fluctuation in the longitudinal x component of velocity just ahead of the flame, evaluated in the Eulerian frame in which the turbulence is stationary (and in which the average x component of velocity ahead of the flame is v_T). The time integral of Eq. (4) relates F to properties of the upstream turbulence for these flows. Thus, if the streamwise Eulerian displacement of fluid elements by the turbulence is defined as $a = \int v' \mathrm{d}t$, Eq. (3) reduces in this approximation to

$$v_T = v_0 \overline{\sqrt{1 + \left(\frac{\partial a}{\partial y} \right)^2 + \left(\frac{\partial a}{\partial z} \right)^2}} \tag{5}$$

Equation (5) indicates that for the flow under consideration the average area ratio a_f/a involves an average of the transverse gradient of the streamwise Eulerian displacement. The Eulerian displacement defines the extent of motion of the flame sheet under the current approximations; the transverse gradient of this provides the differences in the extent of longitudinal motion of adjacent transverse locations of points on the sheet. These differences are responsible for the increased area of the flame sheet. Although the Eulerian displacement is not likely to be statistically stationary, its transverse gradients are, so that v_T is predicted to be independent of time. Equation (4) and therefore the derivation of Eq. (5) apply only if $\partial a/\partial y$ and $\partial a/\partial z$ are small; otherwise, the formula for v_T obtained from Eq. (3) is more complicated[17] and it becomes more difficult to relate v_T to properties of the upstream turbulence. Thus, there are currently outstanding problems in obtaining useful formulas for v_T that are fully justified at the larger values of $|\nabla F|$. In Eqs. (4) and (5), v' and a refer to "near-field" turbulence properties (those just ahead of the wrinkled flame) downstream from a "far-field" region of significant hydrodynamic adjustment. There is current research on relating the near-field properties to far-field properties evaluated in the approaching combustible mixture.[8]

The statistical properties of $\partial a/\partial y$ and $\partial a/\partial z$, needed for evaluating the average in Eq. (5), are often unavailable. Further approximations may be introduced to express these quantities in terms of more readily available turbulence properties. If the near-field upstream turbulence is isotropic, then statistics of transverse derivatives of longitudinal velocity fluctuations are the same as those of longitudinal derivatives of transverse velocity fluctuations. With a Taylor hypothesis, the time integral of the longitudinal derivative of a fluctuating quantity is approximately $1/v_T$ times the fluctuating quantity. Hence, in isotropic turbulence the statistics of $\partial a/\partial y$ are approximately the same as those of a transverse velocity fluctuation divided by v_T, which in turn (again from isotropy) are the same as those of the nondimensional longitudinal fluctuation v'/v_T. Thus, Eq. (5) may be approximated as

$$v_T = v_0\overline{\sqrt{1+2(v'/v_T)^2}} \tag{6}$$

which may be evaluated if a probability-density function for v'/v_T is known.

An idea of the implications of Eq. (6) may be obtained by bringing the average inside the square root. For isotropic turbulence, $\overline{v'^2}=2q/3$, where q refers to the upstream near field; an attempt may be made to take into account departures from isotropy (and possibly also inaccuracy of the Taylor hypothesis) by inserting into this relationship an additional positive factor C, the value of which is unknown. Then Eq. (7) becomes $v_T=v_0\sqrt{1+2C(2q/v_T^2)/3}$, which gives

$$\frac{v_T}{v_0}=\left\{\frac{1}{2}\left[1+\sqrt{1+\frac{8}{3}\,C\left(\frac{2q}{v_0^2}\right)}\right]\right\}^{1/2} \tag{7}$$

In a rough sense, C might also be viewed as accounting for hydrodynamic adjustment so that q in Eq. (7) can refer to the more readily accessible upstream far field.

At small near-field values of $\sqrt{2q}/v_0$, Eq. (7) predicts that v_T-v_0 is proportional to $(\sqrt{2q}/v_0)^2$. This quadratic dependence is consistent with the low-intensity limit of the formula $(v_T/v_0)^2=1+C(2q/v_0^2)$, deduced through a simple physical reasoning by Shchelkin[18] and also derived by Tucker[19] (who ignored the possibility of hydrodynamic instability) as well as in an analysis of a model problem involving a low-intensity expansion.[20]

It may be noted, as seen explicitly in Eq. (6), that v_T is predicted to be independent of turbulence scale in the large-scale regimes under consideration; it may depend on ratios of transverse and longitudinal scales, for example, but not on the magnitude of any one scale. According to Shchelkin,[18] Zel'dovich first deduced this result from dimensional reasoning in unpublished work. There are data that tend to support this prediction.[21] Although the turbulent flame thickness increases with scale (in proportion to the scale at large ℓ_k/δ), the first scale-dependent correction[16,17,22] to the turbulent flame speed is proportional to $(\delta/\ell_k)^2$. When the intensities are low enough to permit Eq. (3) to be used in the first approximation, it is found[17] that the second approximation, to include scale effects, is

$$v_T=v_0\left(\overline{\sqrt{1+|\nabla F|^2}}-(\overline{\kappa^2\sqrt{1+|\nabla F|^2}})g(R_0,\beta,Le_1)\right) \tag{8}$$

where κ (defined below) is a nondimensional flame stretch proportional to δ/ℓ_k and the arguments of the function g are the ratio R_0 of the unburned to burned gas densities, the Zel'dovich number β (also defined below), and the Lewis number Le_1 of the limiting reactant. Thus, influence of turbulence scales on v_T arise through second-order terms involving modifications of structures of the wrinkled laminar flames. To date, the function $g(R_0,\beta,Le_1)$ has been calculated only in the limit of negligible density change and only for flames in which diffusive thermal effects are stabilizing $(Le_1-1>-2/\beta)$, in which case it was found to be[16,22]

$$g(1,\beta,Le_1)=\beta(Le_1-1)[1+\beta(Le_1-1)/8] \tag{9}$$

This formula indicates that modifications of the wrinkled flame structure by the turbulence tend to decrease v_T if $Le_1>1$ and to increase it if $Le_1<1$. It seems likely[16] that $g(R_0,\beta,Le_1)$ will decrease in magnitude with increasing R_0 and typically will produce only small effects on v_T for flames of practical interest, although no experimental tests of these predictions have been made.

Effects of Strain and Curvature on Laminar Flames

Laminar flames in turbulent flows are subjected to strain and develop curvature as consequences of the velocity fluctuations. These influences modify the internal structure of the flame and thereby affect its response to the turbulence, giving rise to the correction exhibited in Eq. (8). In recent years, there has been extensive research on strain and curvature effects; our understanding of them has been improved appreciably by asymptotic methods.[8] Therefore, some discussion of the subject seems warranted here.

Given any velocity field $\boldsymbol{v}$ for a fluid, the motion in the vicinity of a point on the reaction sheet can be resolved into a uniform translation with velocity $\boldsymbol{v}$, a rigid-body rotation with angular velocity $\frac{1}{2}\nabla\times\boldsymbol{v}$ and a pure straining motion.[23] The first two of these motions have no effect on the internal structure of a locally planar reaction sheet. The third is described by the rate of strain tensor $\Phi=\frac{1}{2}[(\nabla\boldsymbol{v})+(\nabla\boldsymbol{v})^T]$. There are many ways to decompose the effect of a general Φ into a superposition of simpler flows. In studying the influence of Φ on the local internal structure of a locally planar sheet, stagnation point flow is the relevant element of the decomposition.[12] Two parameters characterize a general, three-dimensional, stagnation point flow.[24,25] One is the normal element of the rate of strain tensor, $b\equiv\boldsymbol{n}\cdot\Phi\cdot\boldsymbol{n}$, where $\boldsymbol{n}$ is a unit vector locally normal to the sheet, and the other is the ratio c of the principal axes of the intersection of the rate of strain quadratic with the plane of the sheet. $[c=(\partial v/\partial y)/(\partial w/\partial z)$ if y and z are coordinates in the direction of the principal axes and v and w are the velocity components in the directions y and z.] Although the second of these parameters affects details of the sheet structure (e.g., distinguishing between two-dimensional and axisymmetric stagnation points), the first exhibits the major qualitative influence. Thus, analyses of flat flames in counterflowing streams for $-\infty<b<\infty$ with at least one value of c are needed for investigating the influence of strain on laminar flames.

The first analysis of a premixed flame in counterflowing streams of reactants and products is that of Klimov,[12] who addressed adiabatic systems with Lewis numbers of unity and one-step chemistry with strongly temperature-dependent rates of heat release, deriving a number of important aspects of the flame behavior. The principal difference in structure from the unstrained flame arises from the variation of the streamwise component of the mass flux with the normal distance through the flame. A decrease in the normal mass flux with distance through the flame ($b<0$) is reflected in a net transverse outflow, called "flame stretch" by Karlovitz et al.[26] Thus, studies of influences of strain on flames are flame stretch studies. A general definition of flame stretch for planar flames is the time derivative of the logarithm of an area of the flame sheet,[2,12] the boundary of the area being considered to move with the local transverse component of the fluid velocity at the sheet. When Φ is evaluated (conveniently) for the constant-density flow of products just behind the flame, a suitable nondimensional flame stretch becomes $\kappa = -b\alpha_\infty / v_\infty^2$, where v_∞ and α_∞ are the normal adiabatic burning velocity and the thermal diffusivity, both evaluated in the burned gas just behind the reaction sheet. Note that κ is roughly $-b$ times a residence time (δ/v_0) in the flame and is also the reciprocal of an appropriate Damköhler number. The nondimensional stretch κ appears in the nondimensional differential equations for the flame structure.

Many further analyses of structures of stretched laminar flames have now been published.[27-39] With one exception,[39] they essentially apply activation energy asymptotics[40] for one-step kinetics; multistep kinetics can have a significant influence on the response of flames to strain[39] through enhanced diffusive losses of reaction intermediaries from the reaction zone. Most of the analyses employ a constant-density approximation, but a few[32,36-38] have taken density variations into account.

For one-step reactions with Lewis numbers of unity in adiabatic systems, the rate of heat release per unit area is decreased by strain[12]; it is unchanged until the reaction zone begins to exhibit a reactive-convective-diffusion balance and thereafter decreases with increasing κ, approaching zero as κ approaches infinity.[36] Nonadiabaticity and Lewis numbers differing from unity modify the rate of heat release per unit area. Consider the nonadiabaticity associated with the temperature of the product stream at infinity T_∞ differing from the adiabatic flame temperature T_{af}. If the product stream is hotter (a superadiabatic condition), then by enhancing heat conduction (through reducing distances over which heat conduction occurs) and by bringing the reactive-diffusive zone closer to the product side of the stagnation point, an increase in κ results in an increase in the flame temperature at the reactive-diffusive zone and thereby increases the reaction rates and the rate of heat release per unit area of the flame.[36] Through this same mechanism, an increase in κ decreases the heat release rate if the temperature of the product stream is below the adiabatic flame temperature (a subadiabatic condition). In this last situation, if the product stream is cold enough, then an abrupt transition in the flame structure occurs when κ is increased past a critical value[36] that depends on the degree of subadiabaticity through a nondimensional parameter which can be taken to be

$$H = (T_{af} - T_\infty)(T_{af} - T_0)E/(R^\circ T_{af}^3) \tag{10}$$

where E is the overall activation energy, T_0 the initial temperature of the cold reactants, and R° the universal gas constant. For $H<3$ the reaction rate varies continuously with κ, but at larger values of H abrupt transitions occur.

The behavior obtained is illustrated in Fig. 2, where μ is the ratio of the rate of heat release per unit area of the strained flame to that of the adiabatic unstrained flame with the same reactants. In Fig. 2 it is seen that, at the larger values of H, there are S-shaped curves such that with increasing strain rates abrupt transitions occur from conditions of relatively rapid rates of heat release to conditions of relatively slow rates. For an experiment in which κ is caused to increase gradually, the transition would be identified as an abrupt extinction event. Thus, at values of H above a limiting value of order unity, there is a critical nondimensional strain rate κ for extinction, which is a function of H. Since κ is the reciprocal of a suitably defined Damköhler number, an alternative statement is that there exists a critical Damköhler number (dependent on H) below which extinction occurs. At the extinction point, the temperature in the reaction zone has decreased to a value at which the rate of heat release no longer is sufficient for an appreciable amount of reaction to occur in the residence time available in the reaction zone. If in an extinguished state κ is gradually decreased, then there is a different critical value below which ignition occurs (Fig. 2).

The abruptness of the transition in Fig. 2 at larger H is a consequence of the strong dependence of the reaction rate on temperature. At a fixed extent of nonadiabaticity [say, $(T_{af} - T_\infty)/T_{af}$ fixed], a decrease in the Zel'dovich number,

$$\beta \equiv (T_{af} - T_0)E/(R^\circ T_{af}^2) \tag{11}$$

produces a decrease in H and therefore a tendency away from abruptness. For all values of H, it is found that μ approaches zero gradually as κ approaches infinity[36] (subsequent to the abrupt transition, if it occurs); this gradual approach begins at values of κ that are roughly of order β^2 (not shown in Fig. 2, which has been obtained from an analysis that treats κ as being of order unity). For values of β that are small enough (or for sufficiently small extents of subadiabaticity at any given β, as well as for all superadiabatic conditions), only the gradual variations occur. There are many experimental conditions under which abrupt transitions should be observable, e.g., for reaction sheets in turbulent flames with radiant heat loss.

Analogous behavior is attributable to Lewis numbers differing from unity in adiabatic systems.[34,37] In terms of the Lewis number Le_1 of the reactant in a one-reactant system, the relevant parameter that corresponds to H is

$$J = (Le_1 - 1)(T_{af} - T_0)E/(R^\circ T_{af}^2 Le_1) \tag{12}$$

and a multiple-valued dependence of μ upon κ occurs if $J \gtrsim 12$. The source of the variation of μ with κ can be

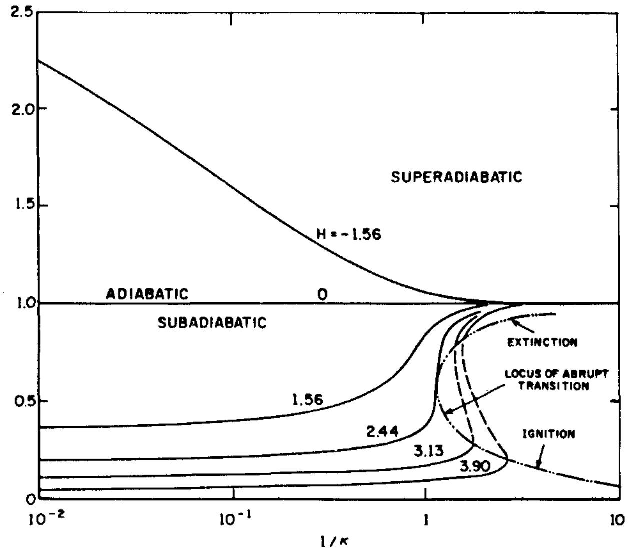

Fig. 2 Ratio of the rate of heat release of a strained flame to that of an unstrained flame as a function of the reciprocal of the nondimensional strain rate for various values of the nonadiabaticity parameter H; calculations were performed for a one-reactant, first-order reaction in an ideal gas with constant specific heat and Lewis number of unity, with thermal conductivity proportional to the temperature and an adiabatic flame temperature four times the initial temperature.[36]

understood physically here on the basis of the influence of Le_1 on the variation of the flame temperature.[37] The characteristics of the abrupt transitions and their interpretations in terms of extinction and ignition parallel those discussed above. The condition $J \gtrsim 12$, needed for abrupt extinctions to occur in adiabatic systems, is encountered for relatively few reactant combinations since it requires reactants with rather high molecular weights or unusually large overall activation energies. Small degrees of subadiabaticity can appreciably enhance the tendency toward the existence of abrupt extinctions[37] with $Le_1 \neq 1$.

The theoretical results that have been discussed here allow κ to be of order unity, a condition best termed one of "moderate strain" (with "strong strain" reserved for large values of κ). Small values of κ are conditions of "weak strain"; under these conditions, the reaction sheet remains far to the reactant side of the stagnation point and, by integrating across the convective-diffusive zone, a formulation in terms of a single partial differential equation for the evolution of the reaction sheet can be derived.[28] By combining the approximations of weak strain and weak curvature, a convenient approach to analyses of wrinkled flames in turbulent flows can be obtained.[8]

Dynamics of Wrinkled Laminar Flames

For weak strain and weak curvature, the influences of strain and curvature phenomena on the flame structure can be characterized in terms of a single quantity, an effective curvature of the flame with respect to the flow, or the total stretch of the flame surface produced by the flow with respect to the moving, curved flame.[17,30,41] By evaluating b from the rate of strain tensor in the products just behind the flame, the last of these quantities may be expressed nondimensionally as

$$\kappa = (\alpha_\infty / v_\infty^2)(-b + v_\infty \nabla \cdot \boldsymbol{n}) \tag{13}$$

Here the curvature term (involving $\nabla \cdot \boldsymbol{n}$) is the stretch that the sheet would experience if it were to move in a quiescent medium at the normal burning velocity v_∞; it is merely necessary to add this stretch associated with the curvature (through the relative motion) to the previously identified stretch associated with the flow in order to obtain the total stretch.

In terms of the reaction sheet function $F(y,z,t)$ introduced earlier, a formula equivalent to Eq. (13) is

$$\kappa = \left(\frac{\alpha_\infty}{v_\infty^2}\right)\left\{\nabla \cdot \boldsymbol{v}_t + \left[v_\infty \nabla^2 F + \left(\frac{\partial}{\partial t} + \boldsymbol{v}_t \cdot \nabla\right)\sqrt{1+|\nabla F|^2}\right]\Big/\sqrt{1+|\nabla F|^2}\right\} \tag{14}$$

where the two-dimensional vector $\boldsymbol{v}_t$ represents the transverse (y,z) components of velocity on the hot side of the flame. The various aspects of geometry and kinematics involved in the derivation of results such as Eq. (14) have been clarified.[17,30,41] There are many other expressions equivalent to Eqs. (13) and (14), e.g., with nondimensionalizations and variables referred to quantities ahead of the flame.

Investigations of the dynamics of wrinkled flames[8] conveniently begin with conservation equations written in a coordinate system moving with the reaction sheet and by appropriate integration across the reactive-diffusive zone remove the chemistry from the field equations, replacing it by suitable jump conditions across the reaction sheet. An expansion for small values of δ/ℓ_k next serves further to separate the problem into near- and far-field subproblems. The domains of the near-field subproblems extend over a characteristic distance of order δ on each side of the reaction sheet. The domains of the far-field subproblems extend upstream and downstream from those of the near-field subproblems over characteristic distance of orders from ℓ_k to ℓ. Thus, the near-field subproblems pertain to the entire wrinkled flame and the far-field subproblems to the regions of hydrodynamic adjustment on each side of the flame in essentially constant-density turbulent flow. Either matched asymptotic expansions or multiple-scale techniques are employed to connect the near- and far-field subproblems. The near-field analysis has been completed for a one-reactant system with allowance made for a constant Lewis number differing from unity (by the amount of order $1/\beta$) for ideal gases with constant specific heats and constant thermal conductivities and coefficients of viscosity[16,17,22]; these results have been extended to ideal gases with constant specific heats and constant Lewis and Prandtl numbers, but with thermal conductivities that vary with temperature.[42] The far-field analysis for turbulent flows has been completed only in a linear approximation that requires $\sqrt{2q}/v_0$ to be small.[8]

The near-field analysis provides expressions for the wrinkled flame motion and the reaction sheet temperature in terms of the flame shape and the gas velocities at the edge of the wrinkled flame. To the first order in the small parameter δ/ℓ_k, this evolution equation for the flame motion may be written as

$$\frac{\partial F}{\partial t} + \boldsymbol{v}_t \cdot \nabla F = v_\ell - v_\infty \sqrt{1+|\nabla F|^2} \times \left\{1 - \kappa\left[I_1 + \frac{1}{2}\beta(Le_1 - 1)I_2\right]\right\} \tag{15}$$

where v_ℓ represents longitudinal velocity component at the same position as $\boldsymbol{v}_t$ (just downstream from the reaction sheet). Integrals appearing in Eq. (15) are

$$I_1 = \frac{R_0}{R_0 - 1}\int_{1/R_0}^{1}\left(\frac{\lambda_1}{e}\right)\mathrm{d}e \tag{16}$$

and

$$I_2 = \frac{R_0}{R_0 - 1}\int_{1/R_0}^{1}\left(\frac{\lambda_1}{e}\right)\ln\left(\frac{R_0 - 1}{eR_0 - 1}\right)\mathrm{d}e \tag{17}$$

where the nondimensional temperature and thermal conductivity are $e = T/T_\infty$ and $\lambda_1 = \lambda/\lambda_\infty$, respectively. If it were desired to measure b and the turbulence velocities in the fresh mixture just ahead of the flame, then $\kappa = (\alpha_0/v_0^2)(-b + v_0 \nabla \cdot \boldsymbol{n})$ would become an appropriate nondimensional total stretch and Eq. (15) again would be obtained in these new variables, but with I_2 now replaced therein by I_2/R_0, and with λ_1 now representing λ/λ_0 in Eqs. (16) and (17). With these new definitions, the nondimensional flame temperature $e_f = T_f/T_\infty$ is found in the first approximation to be

$$e_f = 1 - \kappa(Le_1 - 1)I_2(R_0 - 1)/R_0^2 \tag{18}$$

The term involving κ in Eq. (15) is a correction to the burning velocity v_∞ resulting from the structural modifications to the wrinkled flame produced by stretch and curvature. The corresponding correction to the burning velocity v_0 is somewhat different; the normal velocity of the sheet with respect to the unburnt gas is given by $v_0\{1 - \kappa[I_1 + \frac{1}{2}\beta(Le_1 - 1)I_2/R_0]\}$, with the revised definitions of κ and λ_1.

If κ is omitted and the turbulence velocities are given, then Eq. (15) is a partial differential equation of first order for the evolution of the flame sheet. From Eq. (14) it is seen that, if κ is retained, then the differential equation involves second spatial derivatives. If the analysis were pursued to higher order in δ/ℓ_k, then fourth spatial derivatives would also appear, as has been found[40,43,44] in evolution equations derived on the basis of an approximation of nearly constant density.

Since the turbulence velocities just upstream or just downstream from the wrinkled flame are not given in advance, far-field analyses must be pursued to solve the propagation problem completely. Results of the near-field analysis [specifically Eq. (15) and the corresponding equation involving velocities at the unburnt edge of the flame] and jump conditions for velocities and pressure across the flame[45,46] are needed for approaching the far-field analysis. In the far field, hydrodynamic adjustments arise. In an approximation of nearly constant-density flow, integrations over the far field can be performed to obtain an evolution equation for the flame sheet that includes hydrodynamic effects,[40,43,44] but this approximation is not physically realistic. Ascertaining the influence of the turbulence on the hydrodynamic instability of the laminar flame is a difficult problem that is only beginning to be addressed; even low-turbulence velocities can have suitable phase relationships to negate hydrodynamic instability.[8] The evolution equation may describe wrinkled flame motions of increasing amplitudes in the presence of hydrodynamic instability; for most real flames the diffusive-thermal effects stabilize the short wavelengths of the hydrodynamic instability, wavelengths whose growth would invalidate the approximation of small δ/ℓ_k. Far-field nonlinearities that develop at large amplitudes make the far-field analysis difficult in the presence of hydrodynamic instability. The nonlinearities are present even in the absence of the instability unless $\sqrt{2\overline{q}}/v_0$ is small. For these reasons, very few of the available results include the far-field effects. Thus, these effects are in need of further study.

Turbulent Diffusion Flames

Combustion Regimes

Although v_0 and δ lack physical significance for laminar diffusion flames, a chemical time τ_c for combustion in the hot reaction sheet can be considered. The value of τ_c is more geometry dependent for diffusion flames than for premixed flames because the reactant concentrations in the reaction zone are controlled less by intrinsic laminar flame dynamics. Nevertheless, τ_c may be considered to exist, whence v_0 and δ may be defined from it as artificial parameters. With this roundabout convention, Fig. 1 can be applied to diffusion flames as well, although a skewed plane having R_ℓ and a Damköhler number as coordinates would be interpretable more readily. Damköhler numbers play important roles in classifying regimes of nonpremixed turbulent combustion.[47] That large Damköhler numbers favor a reaction sheet regime is physically evident and also may be seen by viewing Fig. 1 at an angle. Distributed reactions occur at sufficiently small Damköhler numbers. Just as is true in premixed combustion, the boundaries between the regimes are uncertain and are in need of further clarification.

The Relevance of Coupling Functions

In premixed combustion the mixture fraction is a constant and therefore is not involved significantly in theoretical analyses. In turbulent diffusion flames, it varies in space and time and constitutes an especially helpful variable for theoretical analyses, as has been emphasized in the work of Bilger.[47] To define a useful mixture fraction, consider two-stream problems, i.e., those in which the various inlet streams, no matter how complex geometrically, are of two distinct types, one fuel (subscript *F*) and the other oxidizer (subscript *O*), each having a constant (but different) chemical composition and a constant (possibly different) temperature. In a one-step approximation to the chemistry, a fuel oxidizer coupling function may be defined in terms of the local instantaneous mass fractions of fuel Y_F and oxidizer Y_O as

$$X = Y_F/(W_F\nu_F) - Y_O/(W_O\nu_O) \tag{19}$$

where W_F and W_O denote the molecular weights and ν_F and ν_O stoichiometric coefficients, the decrease in the number of moles of F and O in the reaction. Here X is called a coupling function because it couples fuel and oxidizer concentrations and obeys a conservation equation that has no chemical source term—it is also called a conserved scalar; there are many coupling functions or conserved scalars in reacting flows, e.g., the element mole fractions and the total enthalpy. In terms of X, the mixture fraction can be defined as

$$Z = (X - X_O)/(X_F - X_O) \tag{20}$$

so that $Z=1$ in the fuel streams and $Z=0$ in the oxidizer streams. The mixture fraction Z qualifies as a coupling function or conserved scalar and, moreover, has a convenient normalization. Alternative mixture fractions may be defined in terms of element mass or mole fractions or in terms of total enthalpy.

Coupling functions play a central role in reducing problems of turbulent diffusion flames to problems of nonreacting turbulent flows. If reaction rates are sufficiently rapid to maintain chemical equilibrium everywhere, knowledge of the local, instantaneous values of the pressure, enthalpy, and atom mass fractions enable the equations of chemical equilibrium to be used to calculate local, instantaneous values for all of the state variables. In this respect, whenever full chemical equilibrium applies in turbulent combustion, the problem may be expressed as one involving a nonreacting turbulent flow of a fluid with complicated equations of state. Under these conditions, to work with equations involving reaction rates would serve only to introduce potential sources of error; it is better to work with the conserved scalars, for which turbulence modeling has fewer potential sources of error. Influences of the complicated equations of state on appropriate modeling procedures have not been explored very much; the greatly reduced densities and increased molecular diffusivities that tend to prevail near points where the mixture is stoichiometric may affect the best-suitable closure approximations and methods of analysis.

The method of approaching the nonreacting problem must be selected to enable calculations to be made of the quantities of interest for the turbulent flow in chemical equilibrium. The average temperature and the rms temperature fluctuations are examples of quantities that often are of interest. Moment methods for the conserved scalars in the nonreacting problem do not provide this informaiton because the nonlinear dependence of T on Z prevents moments of T from being obtained directly from those of Z. In general, any desired one-point, one-time information concerning temperature and species concentrations may be obtained from the corresponding joint probability-density function for all conserved scalars (pressure, enthalpy, and atom mass fractions). This fact focuses interest on methods that involve probability-density functions. One approach is to select a general class of shapes for the probability-density functions of the conserved scalars, parameterizing the class with a small number of parameters, and to calculate the evolution of the parameters by moment methods applied to the conserved scalars.[47] After the probability-density function $P(Z)$ is obtained at each point in the flow, it is usually possible to calculate any desired averages involving the temperature and species concentrations at each point, including unmixedness.[48]

The results described here rely on the assumptions that Z is independent of its specific definition and obeys a conservation equation having a single molecular diffusion coefficient

D and that the functions $\rho(Z)$ (gas density) and $D(Z)$ exist; the results for T and the mass fraction Y_i further rely on the assumption that these quantities are unique functions of Z. In principle, the procedures can be generalized by working with all of the conserved scalars (pressure, enthalpy, and atom mass fractions) and by introducing and approximating joint probability-density functions for these. However, difficulties would be encountered, especially through the conservation equation for the total enthalpy containing terms for which turbulence modeling is not well developed. Formal derivations involve neglect of pressure-gradient diffusion, of body force work and diffusion and of Soret and Dufour effects, as well as the assumptions of equal binary diffusion coefficients, negligible buoyancy, low Mach numbers, and negligible radiant energy loss. Also, a negligible effect of the time derivative of the pressure on the enthalpy fluctuations is needed, thereby possibly ruling out transient processes such as those occurring in piston engines. The two-stream configuration, negligible fluctuations of state variables in the feed streams, and adiabatic walls (if they are impermeable to chemical species) are further formal restrictions needed in relating the total enthalpy and Y_i to Z on a space-time resolved basis.

The most important restriction is chemical equilibrium and the second most important is equal diffusivities. How critical each of these is in diffusion flames is a topic to which research recently has been devoted. In sufficiently fuel-rich portions of hydrocarbon-air diffusion flames, the chemical equilibrium approximation is not good, but empirical approaches apparently still can be employed to relate nonequilibrium concentrations uniquely to Z with reasonable accuracy for most species.[49] In addition, the extent to which the burning locally proceeds to CO or CO_2 may vary with the fuel, local stoichiometry, and characteristic flow times; methods to account for this are being developed.[50,51] The theoretical methods that have been applied in studying the validity of the two major approximations are expansions for Lewis numbers near unity[52] and expansions in reaction rate parameters for near-equilibrium flows.[5,6,53]

Flame Heights

The "visible flame length" of turbulent jet diffusion flames is a quantity that has been measured, beginning with the earliest studies in the field.[48] It is logical to identify the flame position as the location of the stoichiometric surface and thereby to use Z directly for calculating flame lengths. Thus, in the flame sheet approximation, the instantaneous flame length at any given time would be given by the maximum value of the axial distance from the jet exit to the point at which $Z = Z_c$, its stoichiometric value, at that time. The average flame length is the average value of this quantity and can be estimated directly from the profile $\bar{Z}$ along the centerline. Thus, setting $\bar{Z} = Z_c$ on the centerline gives an average flame length, without need for calculating probability-density functions.

When closer attention is given to the visible flame length, a number of questions arise. If the flame length is a random variable with a probability-density function denoted by $P(h)$, then it will have an average value $\bar{h} = \int_0^\infty hP(h)\mathrm{d}h$ and a mean square fluctuation, $\overline{h'^2} = \int_0^\infty (h-\bar{h})^2 P(h)\mathrm{d}h$. Since $P(h)\mathrm{d}h$ is the probability that the flame length lies in the range $\mathrm{d}h$ about h, it is quite different from $P(Z)$, which always refers to a given point. In principle, there is no way to calculate $P(h)$ solely from knowledge of the functions $P(Z)$ at all points in the flow. The problem encountered here is a type of zero-crossing problem of stochastic processes and its solution involves considering spatial correlations for Z (little is known about two-point, joint probability-density functions for Z). Thus, even if the flame length is identified with $Z = Z_c$, interesting statistical aspects of it lie beyond current calculational abilities. Moreover, there are fundamental uncertainties in relating visible flame lengths to the criterion that $Z = Z_c$. In hydrocarbon flames, blue emissions of radiation typically occur near $Z = Z_c$, but often the blue is obscured by yellow emissions from soot in fuel-rich regions that provide the dominant contributions to the radiant energy flux. The yellow observed need not correlate so well with $Z = Z_c$. Results of measurements of visible flame lengths depend to a degree on the method of measurement, e.g. (with some photographic methods), on the exposure time of a photographic plate. In view of these various uncertainties, inaccuracies may be anticipated in comparisons of measured and calculated visible flame lengths of turbulent diffusion flames.

Production of Trace Species

Oxides of nitrogen and soot-related species are examples of chemical components present in very low concentrations in turbulent diffusion flames; they are trace species. Trace species that maintain chemical equilibrium may be handled directly by the method employing the mixture fraction. However, the trace species of interest are often far from equilibrium. The fact that concentrations of these species are low means that they affect the thermochemistry to a negligible extent and that, therefore, the finite-rate effects for them can be analyzed more easily than those for major species. Methods of analysis have been developed in the literature.[2,5,6,54,55] Activation energy asymptotics often are useful in these methods because the rates of production of the trace species typically depend strongly on temperature.

Average Rates of Heat Release

The pointwise calculation of the average rates of heat release poses special problems in near-equilibrium flows. As equilibrium is approached, the chemical production terms in the equations for species conservation become indeterminate, involving differences of large numbers that cancel. A more circuitous route than the direct use of the rate expressions is therefore needed to find the average rate of heat release. The procedure, developed by Bilger, involves asymptotic expansions and has been described in the literature.[2,6,47] Knowledge of $P(Z)$ is insufficient; it is found that a joint probability-density function for Z and for $\chi \equiv 2D\nabla Z \cdot \nabla Z$ must be known. The non-negative quantity χ is closely related to the instantaneous scalar dissipation rate. Alternatively, since D is a function of Z (through its dependence on T and Y_i), the joint probability-density function for Z and the magnitude of its gradient can be introduced.[2] A need for experimental data on the joint probability-density function remains, as a means for checking the hypothesis of statistical independence often employed in calculations. In addition, more research remains to be done on the asymptotic analysis[53] directed toward obtaining more convenient expressions for higher-order terms.

Effects of Strain on Reaction Sheets

The effects of strain on reaction sheets of laminar diffusion flames have been clarified through activation energy asymptotics in the pioneering work of Liñán.[56] A recent review is also available.[57] Most of the work treats the chemistry in a one-step approximation; there is certainly interest in further study of the influences of strain on multistep kinetics. Peters[58,59] identified a transformation that enables Liñán's results to be applied fully to reaction sheets in turbulent diffusion flames. Thus, a number of conclusions may be drawn concerning the structure and dynamics of turbulent diffusion flames in the reaction sheet regime.

It is found that an increase in $|\nabla Z|^2$ produces an increase in the magnitude of the local, instantaneous, diffusion-controlled reaction rate. Within the context of activation energy asymptotics, this increase terminates abruptly with extinction at a critical maximum value beyond which the chemistry is not fast enough to keep pace with the rate at which the reactants are diffusing into the reaction zone.

Since $|\nabla Z|^2$ is proportional to the local, instantaneous rate of the dissipation of the conserved scalar, which in turn is proportional to the square of a strain rate in the turbulent flow, sufficiently large strain rates cause local reaction sheet extinctions (so long as the temperature of neither inlet stream exceeds the adiabatic flame temperature).

Criteria for local extinction are readily stated from the results of Liñán.[57,59] These criteria introduce restrictions that must be satisfied if the turbulent flame is to be composed of an ensemble of reaction sheets. What happens at higher strain rates is not known. Local extinctions must be followed by a certain degree of nonreactive molecular mixing of fuel and oxidizer. If the conditions are favorable, then the mixed reactants may ignite and burn in a premixed fashion. It has been seen above that, for the premixed combustion to occur in thin fronts, again the strain rate cannot be too large. Perhaps, premixed reaction sheets never can develop subsequent to extinction of reaction sheets in diffusion flames. The overall residence times available in the flow may be too short for the mixed reactants to burn at all. Another alternative is that combustion and heat release of the molecularly mixed reactants occur in broad zones, in a roughly homogeneous manner, like that of a stirred reactor. This mode of burning would be favored in the presence of generally high overall levels of temperature because of the strong dependence of typical reaction rates on temperature; since the rates would then accelerate as heat is released, processes akin to homogeneous thermal or branched-chain explosions may then be envisioned to occur. Bases for specifying conditions for relevance of any of these processes currently are unavailable. There are many studies that could help to improve our understanding of processes that may occur subsequent to reaction sheet extinction. For example, time-dependent analyses of the dynamics of extinction and analyses of the evolution of a hole in a continuous, planar, diffusion flame reaction sheet would provide helpful information.

Liftoff and Blowoff of Turbulent-Jet Diffusion Flames

A working hypothesis that seems reasonable for many turbulent diffusion flames, notably for those in open environments with cold reactants that release large amounts of energy over narrow ranges of stoichiometry at rates strongly dependent on temperature, is that combustion is not reestablished after reaction sheet extinction. This hypothesis can be employed to investigate liftoff and blowoff of turbulent-jet diffusion flames.[58-60] As the velocity of the fuel issuing from the jet is increased, the average flame height in the turbulent range changes little. At sufficiently low fuel velocities, the base of the flame extends to the mouth of the duct, where it is stabilized. When a critical exit velocity (the liftoff velocity) is exceeded, the flame abruptly is detached from the duct and acquires a new configuration of stabilization in which combustion begins a number of duct diameters downstream. The axial distance from the duct exit to the plane at which the flame begins (i.e., the liftoff height of the lifted diffusion flame) increases as the exit velocity is increased further. When the exit velocity exceeds a second critical value (the blowoff velocity), the flame can no longer be stabilized in the mixing region and combustion ceases (blowoff occurs).

The occurrence of liftoff may be understood in terms of critical strain rates for reaction sheet extinction.[59] As the exit velocity is increased, the average local strain rate increases, thus causing an increase in $\bar{\chi}_c$ (the conditioned-averaged χ conditioned on $Z=Z_c$). An increasing fraction of the reaction sheets then encounters extinction conditions, $\chi \geq \chi_E$, where χ_E is the value of $2D|\nabla Z|^2$ at extinction. When too many of the reaction sheets are extinguished, a network of sheets no longer is connected to the burner and liftoff must occur. In a first, rough approximation, this may be assumed to happen when $\bar{\chi}_c \geq \chi_E$. Since χ_E is calculable from thermochemical and chemical kinetic parameters alone, the liftoff criterion expresses a comparison between a fluid-mechanical quantity ($\bar{\chi}_c$) and a chemical property of the system. Thus, $\bar{\chi}_c$ is relevant to liftoff phenomena as well as to average rates of heat release.

After liftoff occurs, the lower temperature at the duct exit produce lower molecular diffusivities, i.e., higher Reynolds numbers, so that $\bar{\chi}_c$ is increased appreciably in the vicinity of the exit. Farther downstream, as the jet spreads, $\bar{\chi}_c$ is reduced. Beyond some distance from the duct exit, the condition $\bar{\chi}_c \leq \chi_E$ is again established and the turbulent diffusion flame may exit. The liftoff height then may be determined[59,60] as the height at which $\bar{\chi}_c = \chi_E$. This height increases as the jet velocity increases because, at any fixed height, $\bar{\chi}_c$ increases with the jet velocity. Although there are significant uncertainties in estimating $\bar{\chi}_c$ for turbulent jets, reasonable agreements between measured and estimated liftoff heights have been obtained.[59,60]

An alternative way to calculate liftoff heights is to set the local average flow velocity in the jet equal to a premixed turbulent flame speed.[61-63] This is done on the basis of the assumption that thorough molecular scale mixing occurs in the cold jet upstream from the point of flame stabilization. The universal validity of this assumption is questionable and in need of further study.[59] If it is a reasonable approximation, then associated with the radial mixture ratio profile of the nonreactive jet at each axial location is a flame speed profile that, when balanced against the average velocity profile through identification of an axial distance which results in a tangency condition, gives the flame base position for each exit velocity. Further research may help to establish ranges of validity for the two alternative theories.

Conclusions

From this review, it is seen that in recent years asymptotic methods have contributed greatly to an improved understanding of turbulent combustion in both premixed and nonpremixed systems. These methods must be incorporated into flowfield calculations before they can be fully exploited. A number of outstanding questions have been identified in the course of this review. Further pursuit of asymptotic methods to address these questions may lead to advances in our abilities to predict the processes of turbulent combustion.

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